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Tests for Coefficient of Variation

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SUMMARY

In the field of agriculture, zoology, genetics and other allied fields, the coefficient of variation is widely used. Howe· ·, no standard parametric test procedures were found to be in use to test the C.V.s. Therefore, using different established results, parametric test procedures were developed and presented in this note for their application in various disciplines. Their robustness and the small sample behaviour are also discussed.

Key words : coefficient of variation, Wald test, maximum likelihood estimator, jackknife test, bootstrap test.

Introduction

In the disciplines of agriculture, zoology, genetics and other allied fields, coefficient of variation (C.V.) is a widely used measure of dispersion. But, a close observation of scientific articles reporting the C.V. s in various journals reveals that they are not supported by appropriate tests. Although many of the standard text books in statistics/applied statistics, discuss at length various parametric and non parametric tests, generally they are silent about tests on C.V. Hence, there may be a general impression that tests on C.V. do not exist.

Although small sample tests on C.V. do not exist, the asymptotic standard error of C.V. is known in the theoretical field (Kendall and Stuart [5]; Serfling, [11]). Using the standard error, a large sample test on C.V. can be constructed. However actual construction of test procedures based upon the standard error is beyond the purview of the applied scientists. The aim of this note is to present these tests in an explicit form for the benefit of scientists in their applied research. It also presents computer oriented test procedures, namely jackknife tests and bootstrap tests on C.V. These tests are described in detail in the sections 2 and 3. The robustness and small sample behaviour of the tests are in section 4 and the note concludes in section 5.

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2. *Large Sample Classical Tests on C.V.*

Let a sample of size n be taken from a normal population with mean μ (which is not equal to zero) and variance σ^2; $\overline{X}$ and S^2 denote the sample mean and variance. Then to test the null hypothesis :

$$H_0^{(1)} : \frac{\sigma}{\mu} = \Delta_o \quad \text{(specified)}$$

the test statistic is

$$Z_1 = \frac{\sqrt{n}(S/\overline{X} - \Delta_o)}{[(S/\overline{X})^4 + 1/2\,(S/\overline{X})^2]^{\frac{1}{2}}} \tag{2.1}$$

When the sample size is large, the asymptotic distribution of Z_1 under H_0 is standard normal, and decision can be taken using critical values of the normal distribution.

When independent sample of size n_1 and n_2 are taken from two normal populations with means μ_1 and μ_2, variances σ_1^2 and σ_2^2, then to test the null hypothesis $H_0^{(2)} : \frac{\sigma_1}{\mu_1} = \frac{\sigma_2}{\mu_2}$, the test statistic is

$$Z_2 = \frac{(S_1/\overline{X}_1 - S_2/\overline{X}_2)}{\left[\sum \frac{1}{n_i}\left((S_i/\overline{X}_i)^4 + \frac{1}{2}(S_i/\overline{X}_i)^2\right)\right]^{\frac{1}{2}}} \tag{2.2}$$

where $\overline{X}_i$, S_i^2, $i = 1, 2$ denote the sample mean and variance respectively. Again when the sample sizes are large, the asymptotic distribution of Z_2 under H_0 is N(0, 1).

The validity of the tests is indicated by the asymptotic properties of maximum likelihood estimators. Also Z_1 and Z_2 are the Wald test statistics for the respective hypothesis under consideration. The proof is straightforward.

Remark 1 : The standard errors reported are quite general in nature, based on four central moments of the distribution. Thus one can construct a test on C.V. for any distribution.

Remark 2 : The results can be extended concerning two populations for testing the equality of C.V.'s of k populations. The statistic is

$$X_0^2 = \sum [(S_i/\overline{X}_i) - (S/\overline{X})_w]^2 / V_i, \tag{2.3}$$

where $$(S/\overline{X})_w = \sum_{i=1}^{k} \frac{1}{V_i}\left(\frac{S_i}{\overline{X}_i}\right), \text{ the pooled estimate of C.V.}$$

and $$V_i = (S_i/\overline{X}_i)^4 + 1/2\,(S_i/\overline{X}_i)^2.$$

Under H_o, the asymptotic distribution of X_0^2 is chi-square with k-1 d.f.

3. *Computer Oriented Tests*

3.1 Jackknife Test :

The test in the case of a single sample is as follows :

For notational convenience, denote $\theta = \sigma/\mu$ and $\hat{\theta} = \frac{S}{\overline{X}}$. Let $\hat{\theta}_{-i}$ denotes the sample coefficient of variation deleting the ith observation. Define

$$\hat{\theta}_i = n\,\hat{\theta} - (n-1)\,\hat{\theta}_{-i}, \quad i = 1, \ldots, n$$

and $$\tilde{\theta} = \frac{1}{n}\sum \hat{\theta}_i$$

To test $H_0^{(1)}$, define the test statistic

$$Z_3 = \frac{\sqrt{n}(\hat{\theta} - \Delta_o')}{\left[\frac{1}{n-1}\sum (\hat{\theta}_i - \tilde{\theta})^2\right]^{\frac{1}{2}}}, \tag{3.1}$$

which is asymptotically normal, under $H_0^{(1)}$, with mean 0 and variance 1 (Reeds, [10]).

3.2 Bootstrap Test :

The procedure for a single sample test is outlined here. Generate a random sample of size n from a normal distribution with mean $\overline{X}$ and variance S^2. Let us denote the mean and variance for this generated sample by $\overline{X}^*$ and S^{*2}. Analogous to Z_1 define,

$$Z_1^* = \frac{\sqrt{n}\,[(S^*/\overline{X}^*) - (S/\overline{X})]}{[(S^*/\overline{X}^*)^4 + 1/2(S^*/\overline{X}^*)^2]^{\frac{1}{2}}} \tag{3.2}$$

Let Z_α^* be the α percentile of the Bootstrap distribution of Z_1 that is Z_1^*, based on 1000 simulations. Then the bootstrap test procedure consists of rejecting H_0 if (C. f. 2.1) > Z_α^* (see Hall and Wilson [2] regarding bootstrap tests in general).

Remark 3 : The standard error of C.V. reported in Kendall and Stuart [5] and Serfling [11] do not pertain to any specific distribution. Hence, the same procedure outlined above can be applied to develop non-parametric bootstrap test also.

4. *Small Sample Behaviour and Robustness of the Tests*

Even when the sample is taken from a normal population, for small samples the normal approximation to S^2 is rather poor. To improve the approximation of Z_1 and Z_2, Rao and Bhatt [8] obtained Edgeworth expansion of the null distribution of the sample coefficient of variation to the order of $0(n^{-2})$. Using this more accurate P value can be obtained. For large samples the normal approximation for the null distribution of the classical tests on C. V. may be fairly accurate. For small samples however, the modification detailed above though a little cumbersome, is better suited. Where computer facility is available along with a good programming assistance, use of bootstrap tests is advocated.

Rao and Vidya [9] studied the robustness of the tests Z_1 and Z_2. They found that the single sample test Z_1 is fairly robust even if the observations do not come from a normal distribution. This is a highly useful and significant observation in many of the applications of the tests. Further they also observed that the two sample test Z_2 is also robust when the samples are moderately correlated. Study on the small sample behaviour and robustness of computer oriented tests on C.V. are not available at present.

5. *Conclusion :*

The tests on C.V. are explicit enough so as to be easily adopted in applied research works. Since these test procedures are not readily available in the test books on statistics, the present note, thus bridges a gap between the theoretical and applied aspects of scientific pursuit.

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A Class of Unbiased Ratio Estimators

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SUMMARY

A class of unbiased ratio-type estimators for population mean $\bar{Y}$ is defined based on a linear combination of three estimators viz: $\bar{y}$, $(\bar{X}/n)\sum_{i=1}^{n}(y_i/x_i)$ and $(\bar{x}/n)\sum_{i=1}^{n}(y_i/x_i)$. It is shown that Hartley and Ross [1] estimator is a particular member of the class. Exact variance of the proposed class is derived. The optimum estimator in the class in the minimum variance sense is identified. Two numerical examples are included to illustrate the results.

Key words : Unbiased ratio estimator, optimum estimator, minimum variance.

Introduction

Consider a finite population with units $U_1, U_2, \ldots, U_N$. For simplicity let the variate of interest y and the auxiliary variate x related to y assume real non-negative values (y_i, x_i) on the unit U_i, $i = 1, 2, \ldots, N$. We are interested in estimating population mean $\bar{Y}$ utilising information on $\bar{X}$, the population mean of auxiliary character x.

Let y_i and x_i denote respectively the y and x values for the ith sampled unit, $i = 1, 2, \ldots, n$ in simple random sampling without replacement (SRSWOR). Let $r = \bar{y}/\bar{x}$ and $\bar{r} = (1/n)\sum_{i=1}^{n}(y_i/x_i)$ where $\bar{y}$ and $\bar{x}$ are respectively sample means of y and x variates. In SRSWOR Hartley and Ross [1] proposed the following unbiased estimator for $\bar{Y}$:

$$\bar{y}_{HR} = \bar{r}\bar{X} + (n(N-1)/N(n-1))(\bar{y} - \bar{r}\bar{x}) \qquad (1.1)$$

or, equivalently

$$\bar{y}_{HR} = \bar{r}\bar{X} + ((N-1)/N)\ s_{rx} \qquad (1.2)$$

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